

AP Calculus AB

Volume by Rotation

1) Disc - dx

$$V = \pi \int_{-3}^0 (2\sqrt{x+4})^2 dx$$

3) WASHER - dx

$$R = -x^2 + 4 - 1 \quad r = 3 - 1 = 2$$

$$V = \pi \int_{-1}^0 (-x^2 + 3)^2 dx - \pi \int_{-1}^0 4 dx$$

2) Disc - dy

$$V = \pi \int_0^1 (-y^2 + 1)^2 dy$$

4) WASHER - dy

$$R = -y^2 + 7 - 1 \quad r = 3 - 1 = 2$$

$$V = \pi \int_{-2}^2 (-y^2 + 6)^2 dy - \pi \int_{-2}^2 4 dy$$

5) Disc - dy

$$R = y^2 - 0$$

$$V = \pi \int_0^4 y^4 dy$$

$$\pi \left[\frac{1}{5} y^5 \right]_0^4$$

$$\boxed{\frac{32\pi}{5}}$$

6) WASHER - dx

$$R = 3 \quad r = \frac{1}{x} \quad \frac{1}{x} = 4 \quad x = \frac{1}{4}$$

$$V = \pi \int_{\frac{1}{3}}^4 3^2 dx - \pi \int_{\frac{1}{3}}^4 \left(\frac{1}{x}\right)^2 dx$$

$$= \pi \left[9x \right]_{\frac{1}{3}}^4 - \pi \int_{\frac{1}{3}}^4 x^{-2} dx$$

$$\pi \left[33 \right] - \pi \left[-x^{-1} \right]_{\frac{1}{3}}^4$$

$$33\pi - (\pi(-\frac{1}{4} - (-3)))$$

$$\boxed{33\pi - \frac{11}{4}\pi}$$

7) disc - dy

$$R = -y^2 + 5 - 1 = -y^2 + 4$$

$$V = \pi \int_{-2}^2 [-y^2 + 4]^2 dy$$

OR

$$V = 2\pi \int_0^2 [-y^2 + 4]^2 dy$$

$$V = 2\pi \int_0^2 (y^4 - 8y^2 + 16) dy$$

$$2\pi \left[\frac{1}{5} y^5 - \frac{8}{3} y^3 + 16y \right]_0^2$$

$$2\pi \left[\frac{32}{5} - \frac{64}{3} + 12 \right]$$

8) WASHER - dx

$$R = -x^2 + 3 \quad r = 2$$

$$V = 2\pi \int_0^1 (-x^2 + 3)^2 dx - 2\pi \int_0^1 4 dx$$

$$V = 2\pi \int_0^1 (x^4 - 6x^2 + 9) dx - 2\pi \int_0^1 4 dx$$

$$= 2\pi \left[\frac{1}{5} x^5 - 2x^3 + 9x \right]_0^1 - 8\pi$$

$$= 2\pi \left[\frac{1}{5} - 2 + 9 \right] - 8\pi$$

$$= \frac{2\pi}{5} + 6\pi = \frac{28\pi}{5}$$

9) WASHER - dy

$$R = -y^2 + 7 \quad r = 3$$

$$V = \pi \int_0^1 (-y^2 + 7)^2 dy - \pi \int_0^1 9 dy$$

$$= 111.631$$

11) WASHER - dx

$$R = 2 - (-3) \quad r = 2 - (\sqrt{x} - 3)$$
$$= 5 \quad r = 5 - \sqrt{x}$$

$$V = \pi \int_0^4 25 dx - \pi \int_0^4 (5 - \sqrt{x})^2 dx$$

$$= 142.419$$

10) WASHER - dx

$$R = \sqrt{x} + 1 - (-2) \quad r = x^2 + 1 - (-2)$$
$$= \sqrt{x} + 3 \quad = x^2 + 3$$

$$V = \pi \int_1^2 (\sqrt{x} + 3)^2 dx - \pi \int_1^2 (x^2 + 3)^2 dx$$

$$= 7.226$$

12) WASHER - dx

$$R = -2 - (x^2 - 5) \quad r = -2 - (x^2 - 3)$$
$$= 7 - x^2 \quad = 1 + x^2$$

$$V = \pi \int_0^1 (7 - x^2)^2 dx - \pi \int_0^1 (x^2 + 1)^2 dx$$

$$= 16.755$$